

Probing the Boundary of the Non-Perturbative QCD by Small Size Instantons

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The lattice data on the instanton size distribution suggest an additional $O(\rho^2)$ action. The small- ρ effect $O(\rho^4)$ predicted by the Operator Product Expansion (OPE) is not observed. Similar deviations are also found for non-perturbative response to a small static color dipoles: it is $O(r)$ not $O(r^2)$. We suggest that small instanton radii in the QCD vacuum and small radii of the QCD strings (to which this observation relates) are consequences of the same phenomenon: a very robust dual superconductivity in the QCD vacuum, with relatively large Higgs VEV and surprisingly large Higgs mass.

1. While perturbative treatment points toward the $\Lambda_{QCD} \sim 200 \text{ MeV}$ as the momentum scale where it becomes inapplicable, it is well known by now that the non-perturbative phenomena actually turn on at larger momenta. Where it happens depends on a particular physical problem considered: at least *three different scales* have been identified so far.

The first is the so called *chiral scale* $\Lambda_\chi \sim 1 \text{ GeV}$, the upper limit of low energy effective theories such as chiral effective Lagrangians or Nambu-Jona-Lasinio model [1] (and it is thus the oldest one, identified already in 1961). Its other incarnation is a momentum scale at which QCD sum rules for scalar and pseudoscalar channels fail [2].

The second scale, identified two decades later [2], is larger $\Lambda_{0\pm \text{glueballs}} \sim (3-4) \text{ GeV}$. It is defined as momenta at which spin zero *gluonic* correlation functions (of operators $G_{\mu\nu}^2$ and $G_{\mu\nu}\tilde{G}_{\mu\nu}$) deviate from their perturbative behavior. (Note: it is *not* the glueball masses!)

The physical origin of both these scales has been traced down to *instantons-induced effects* [10]. For recent detailed review of these issues see [3], for more recent comparison between the QCD and the Seiberg-Witten solution for the $N=2$ supersymmetric theory see [4].

2. This letter is devoted to the *third* non-perturbative scale, related with the onset of confinement forces. We use instantons again, but only as a small probe: other small probes should show similar effects. Thinking about confining forces at *small* distances may appear strange: it is indeed true that their manifestation at large distances is more important. Nevertheless, we study the non-perturbative corrections to properties of small-size “color dipoles”, of three different kinds.

Historically the first example is states of heavy quarkonia. The non-perturbative correction to their energies was calculated by Voloshin and Leutwyler [5] by OPE: $\delta E \sim \langle 0|G_{\mu\nu}^2|0 \rangle r^2 \tau$ where the spatial size $r \sim 1/(\alpha_s M)$ and the rotational time $\tau \sim 1/(\alpha_s^2 M)$ for large quark mass M and small α_s . I am not aware of any precision studies of whether in this case it indeed works.

Instantons is another kind of “dynamical dipoles”, now with $r \sim \tau \sim \rho$. In fact, they are much more sensitive tool because the probability of the tunneling events contains the perturbative charge and all corrections in

the exponent. As noticed in [7], it is in particular true for fixing the actual value of Λ_{QCD} : while hadronic masses and quarkonium levels currently used for this purpose are $O(\Lambda_{QCD})$, the density of small size instantons is $O(\Lambda_{QCD})^b$ where $b = (11N_c/3 - 2N_f/3) \sim 10$. (This large power is nothing but the famous one-loop beta function coefficient, N_c, N_f are the number of colors and flavors). So, with comparable accuracy, the instanton-based determination should potentially be 10 times more accurate!

Similar to Voloshin-Leutwyler correction, there is the OPE-based result [6] predicts the following correction to the density of instantons of size ρ :

$$dn(\rho) = dn_{pert}(\rho)(1 + \frac{\pi^4 \rho^4}{2g^4} \langle 0|G_{\mu\nu}^2|0 \rangle + \dots) \quad (1)$$

Note the generic 4-th power of ρ : in QCD it is the dimension of the lowest gauge invariant local operator. Note also the sign: it is nothing else but a generic attraction resulting from *any* second order perturbation. However both these conclusions happen to be in apparent conflict with the lattice data (see below).

Furthermore, recent studies of the vacuum reaction to small-size *static* dipoles (see point 6 below) show similar deviations from the OPE-based expectations. We argue that *both* phenomena can be naturally understood, provided: (i) there is rather high non-perturbative scale, so that (ii) one can use an effective theory rather than QCD itself; (iii) which should include *scalar* composite fields with a non-zero VEV.

3. In Fig.[1](a) we show recent lattice data for the instanton size *distribution* in pure $SU(3)$ gauge theory [8]. (There are others but, this work includes such refinements as improved lattice action and back extrapolation to zero smoothening.) One can clearly see, that a rapid rise at small ρ turns into a strong suppression. The former behavior is consistent with the semi-classical one-loop result [9]:

$$\frac{dN_0}{d\rho}|_{pert} = \frac{C_{N_c}}{\rho^5} \left(\frac{8\pi^2}{g^2(\rho)} \right)^{2N_c} (\rho\Lambda)^b \quad (2)$$

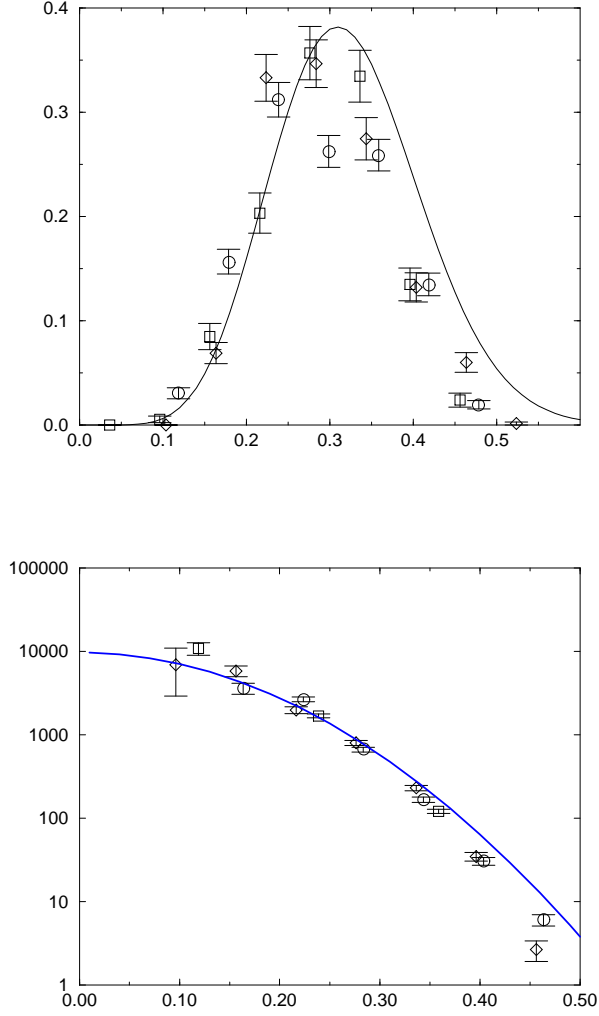


FIG. 1. (a) The instanton density dn/dpd^4z , $[\text{fm}^{-5}]$ versus its size ρ [fm]. (b) The combination $\rho^{-6} dn/dpd^4z$, in which the main one-loop behavior drops out for $N_c = 3, N_f = 0$. The points are from the lattice work [8], for this theory, with $\beta=5.85$ (diamonds), 6.0 (squares) and 6.1 (circles). Their comparison should demonstrate that results are rather lattice-independent. The line corresponds to the proposed expression $\sim \exp(-2\pi\sigma\rho^2)$, see text.

where C_{N_c} is the normalization constant, the ρ^{-5} factor and the term with the coupling constant comes from the Jacobian of the zero modes, and $b = (11N_c/3 - 2N_f/3)$ as already mentioned.

Sharp maximum seen in Fig.1(a) appears at rather small $\rho \approx 1/3$ fm, much smaller than their spacing $R \approx 1$ fm. This results in a non-trivial “vacuum diluteness” parameter [10]

$$(\rho/R)^4 \sim (1/3)^4 \quad (3)$$

which is so instrumental in understanding of many instantons-induced effects. However now we are not interested in a typical instanton but in their suppression,

and so in Fig.1(b) we re-plot the same data, with the leading semi-classical behavior taken away. One can see the same suppression pattern at both sides of the maximum. The OPE prediction (1) is not seen: probably it is only true at smaller ρ . The effect is clearly $O(\rho^2)$, and not just for small ρ but in the whole region.

4. What can be the mechanism of such $O(\rho^2)$ suppression? Before we try to answer, let us recall other suggestions from literature.

Diakonov and Petrov [11] studied the instanton ensemble using the simplest “sum ansatz” for gauge field configurations and found strong repulsion of II and II . It can generate diluteness even stronger than (3), but the result is not robust, other trial functions have different amount of repulsion, with the so called “streamline” configurations [12] having no repulsion for some relative orientations. Furthermore, in dilute instanton ensemble the suppression is different at both sides of the maximum, with the OPE result (1) true for smaller ρ .

In [7] it was suggested that more *slowly* running coupling constant at large distances plus the Jacobian-related factor ρ^{-5} in the instanton measure may be sufficient to make the density at least convergent at large ρ . (The motivation included the well known fact that for sufficiently large number of flavors it “freezes” at a fixed point.) But this effect cannot be there at sizes as small as $\rho \sim 0.1$ fm, and it is not leading to the $\exp(-O(\rho^2))$ law.

5. The central idea of this Letter is that $O(\rho^2)$ suppression of instantons is due to a “dual superconductivity” [14], a scenario in which some composite objects condense, forming the non-zero vacuum expectation value (VEV) of the magnetically charged scalar field ϕ . My first (naive) argument was that in such theory, unlike the QCD itself, at least there is the dimension-2 operator $|\phi|^2$.

The composites may be magnetic monopoles [14,15], or P-vortices, or something else: anyway one is lead to an incarnation of the old Landau-Ginzburg effective theory, Abelian Higgs Model (AHM), describing interaction of a “dual photon” and “dual Higgs” fields. AHM was applied to the description of the QCD strings, as Abrikosov-Nielsen-Olesen vortices in [16].

Before we go into details, let us point out a striking similarity between these two problems. A vortex is the 2d topologically non-trivial configuration, in which ϕ vanishes at the center, the Dirac string where the dual potential is singular. An instanton problem is in a way the previous one squared. The 4d picture of the fields is like two string cross sections in two orthogonal 2d planes. Higgs field ϕ again vanishes at the center, because in the singular gauge (the only one good for multi-instanton configurations) the gauge field is $A_\mu(x)^2 \sim 1/x^2$ at the origin, acting as a centrifugal barrier. Since “melting” of the dual superconductor at the center is not a small modification, one generally cannot expect the OPE-type calculations to hold. In both problems one has first to solve for the field and then calculate the energy or action.

Fortunately, for instantons in a Higgsed vacuum it was already done by 't Hooft [9]: for fundamentally charged Higgs the answer is

$$\Delta S = 4\pi^2 \rho^2 |< \Phi >|^2 \quad (4)$$

Note that it leads to the $O(\rho^2)$ suppression law we need to explain Fig.[1](b), and that ΔS should not necessarily be small. We return to speculations on the exact nature of the Higgs and the dual photon fields of the Landau-Ginzburg model (needed to evaluate the strength of the effect) at the point 7 below.

6. Now we briefly review applications of the dual superconductivity idea to the QCD string, or the ANO vortex line, done in a series of papers [19]. Among clear successes of this approach is: (i) prediction of weak string-string interaction, putting it around the boundary of type I and II superconductivity; (ii) prediction of a whole set of potentials other than central. Both agree well with available lattice data, for a review see [13].

The effective Lagrangian used in [18] is

$$L = \frac{4}{3} [\frac{1}{4} (\partial_\mu C_\nu - \partial_\nu C_\mu)^2 + \quad (5)$$

$$\frac{1}{2} |(\partial_\mu - ig_m C_\mu) \phi|^2 + \frac{\lambda}{4} (|\phi|^2 - |\phi_0|^2)^2]$$

where we have omitted interaction with quarks at the ends. C_μ is dual color potential coupled to Higgs with magnetic coupling $g_m = 2\pi/g$. Assuming that we are exactly at the boundary of the type I and II superconductivity, the masses of the Higgs and the “dual photon” are equal $M_\phi = M_C = g_m \phi_0$. The (classical) string tension is directly related to Higgs VEV

$$\sigma = \frac{4\pi}{3} |\phi_0|^2 \quad (6)$$

Lattice studies of the QCD strings have shown that they are surprisingly thin. According to [13], the “energy radius” (at which it decreases by $1/e$) is about $\delta_{1/e} \approx .18$ fm, while that for the action distribution is about twice larger. In effective dual model [18] the string width is related to masses of dual photon and Higgs, being the *large non-perturbative scale* of the 3-ed kind we are speaking about. The data mentioned put it in the “glueball mass range”, around 1.3 GeV according to [13] (It is difficult for me to access the error involved.)

This observation also has many phenomenological consequences. One is just another argument explaining weak string-string interactions known from Regge phenomenology. Another is “hadron diluteness”: color field inside hadrons occupy only few percent of the volume

$$(\delta_{1/e}/R_h)^2 \sim (1/5)^2 \quad (7)$$

contrary to the MIT bag model which views the *whole* hadronic interior to be in the perturbative phase. In

other words, the value for the bag constant $B_{MIT} \sim 50 \text{ MeV}/\text{fm}^3$ was hugely under-estimated: it is $B \sim 1000 \text{ MeV}/\text{fm}^3$ or more. (Similar but different argument was made two decades ago in [24].)

Another way to look at this issue is to use *small static color dipoles*, or short strings. As time is unlimited there is no OPE prediction like (1), but using the second order dipole approximation [17] instead one gets $O(r^2)$ corrections to the static potential

$$V(r) = -\frac{4\alpha_s(r)}{3} \frac{1}{r}$$

$$+ r^2 \int d\tau e^{(-\frac{3\alpha_s(r)\tau}{2r})} < 0 | G_{\mu\nu}(\tau) U_\tau G_{\mu\nu}(0) U_\tau^\dagger | 0 > \quad (8)$$

where the field strengths are separated by the time delay τ , with U_τ being the appropriate parallel transports. However recent lattice data on $V(r)$ at small r [21] have found a $O(r)$ effect, as suggested previously in [20]

$$V(r) = -\frac{4\alpha_s(r)}{3} \frac{1}{r} + \sigma_0 r + \dots \quad (9)$$

The small-distance tension is larger than the asymptotic one $\sigma_0 \approx (4-5)\sigma_\infty$, but with rather uncertain error from perturbative subtraction. It was shown [22] that $O(r)$ term appears in AHM, although with about the same linear potential at all r , $\sigma_0 \approx \sigma_\infty$.

7. The conclusions we draw from all these observations are: (i) the distances $r = 0.1 - 0.2$ used in these studies are already *large enough* to be outside the validity domain of the OPE; (ii) but an effective model like AHM should rather be used, and (iii) it does provides at least qualitative explanation of the non-perturbative effects.

Encouraged by this, we return to instantons and try to apply the same reasoning. Since both the 't Hooft correction (4) and the string tension (6) scales as the Higgs VEV squared, we expect qualitatively that

$$\frac{dN}{d\rho} = \frac{dN}{d\rho} |_{pert} \exp(-C\sigma\rho^2) \quad (10)$$

where C is some numerical constant. In order to find C one has to identify the scalar and the dual photon fields of the AHM, and explain how they are coupled to the colored gauge field of the instanton. In the AHM treatment of the QCD string [18] the magnetic field of the dual photon is identified directly with the color-electric gauge field inside the string. It does not create problems because this electric field can be considered Abelian.

Applying the same ideas for instantons, let us first note that their *self-duality* helps: because electric and magnetic fields are identical, the “magnetic” potential C_μ and the original one A_μ are also the same. However both are intrinsically non-Abelian, so only a particular component (or a combination of those) can be identified with the Abelian “dual photon” of the effective theory. In other words, an *Abelian projection* is inevitable, and there is no unique or preferred way to do it. Lacking

better ideas, we simply do what lattice people do: just select one of possible projections and see what happens. Clearly, selecting Higgs field interacting with a particular component of the gauge potential means breaking the gauge group (which the vacuum of the Standard model does and that of QCD does not). But we proceed anyway, simply re-scaling the dual fields in a way that their Lagrangian (1) matches the 't Hooft one. If we do so, it leads to identification $\langle \Phi \rangle^2 = (2/3)\phi_0^2$, or the constant C in (10) to be $C = 2\pi$. Putting it all together, we can now compare [23] this result to the exponential suppression. The corresponding curves are shown in Fig.[1], and it works very well.

9. Brief summary. The dual superconductivity seems to be surprisingly robust. Instanton suppression we discussed and string tension considered before both fix the AHS Higgs VEV rather accurately. The string size and small- r potential provide hints that the Higgs and dual photon masses are large, in the 1-2 GeV range. Their nature remains unclear: so the status of AHM Higgs is, ironically, not that different from that of the Standard Model.

Outlook: one may test our suggestions by comparing instanton suppression in theories with variable number of colors and/or flavors. Unfortunately available data for the SU(2) color group or SU(3) with dynamical fermions are not yet good enough to do so.

Another challenging set of questions is related with instanton suppression at non-zero temperatures T and/or densities. At high T or density, in the quark-gluon plasma phase, the answer is clear: the instanton electric fields are again suppressed, but now by the usual Debye screening [24]. It leads to a factor $\exp(-a(T, \mu)\rho^2)$ similar to the one discussed above, where the coefficient $a(T, \mu = 0)$ was calculated in [25] and then generalized to the $\mu \neq 0$ in [10]. Note that confinement is not completely gone [26]. The most interesting point is what happens close to the deconfinement transition. Since 2 and 3 color gauge theories have it of the second and the first kind, respectively, a detailed study of the instanton suppression at $T \approx T_c$ is of great interest.

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